A Note on Supersymmetry Breaking

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Abstract

Using a simple observation based on holomorphy, we argue that any model which spontaneously breaks supersymmetry for some range of a parameter will do so generically for all values of that parameter, modulo some isolated exceptional points. Conversely, a model which preserves supersymmetry for some range of a parameter will also do so everywhere except at isolated exceptional points. We discuss how these observations can be useful in the construction of new models which break supersymmetry and discuss some simple examples. We also comment on the relation of these results to the Witten index.

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1 Holomorphy and SUSY breaking

Recently, due to renewed interest and improved techniques [1], there has been a great deal of progress in our understanding of spontaneous (dynamical) supersymmetry breaking. The number of models which are thought to break SUSY has increased substantially [2, 3, 4, 5, 6] relative what was known a decade ago [7]. In this note we wish to examine the question of SUSY breaking in the parameter space of a specified model with fixed field content and interactions. We will argue that the generic behavior in the whole of parameter space can be deduced from that of any small patch. That is, models either do or do not break SUSY in the entire parameter space, with the possible exception of isolated points in the space which form a set of measure zero. This conclusion is similar to that obtained from the Witten index [8] in the case of $\mathrm{Tr}(-1)^{\mathrm{F}} \neq 0$ – *i.e.* when SUSY is unbroken. However, the picture we obtain extends also to models in which $\mathrm{Tr}(-1)^{\mathrm{F}} = 0$ and the status of SUSY breaking is still ambiguous.

The argument for this is simple and based on holomorphy. Consider the Wilsonian effective action for a model which possesses $N = 1$ SUSY:

$$
\begin{aligned}
\mathcal{L}_{\mathrm{eff}} \;=\;& \int d^4\theta \, K(\Phi^\dagger e^V \Phi) \;+\; \tfrac{1}{8\pi} Im \int d^2\theta \, \tau(\Phi) \, \mathcal{W}^\alpha \mathcal{W}_\alpha \\
&+\; 2Re \int d^2\theta \, W(\Phi) \;+\; \{\text{higher dim. } \mathcal{W}^2\} \,,
\end{aligned}
\tag{1}
$$

with Kähler metric

$$
g_{ij} = \frac{\partial^2 K}{\partial \Phi^{\dagger i} \partial \Phi^j} \,.
\tag{2}
$$

In the last term in (1) we include higher dimension D- and F-terms which depend on $\mathcal{W}^\alpha$ (K and W already include higher dimension operators in Φ_i). For simplicity below we will focus on Φ_i in our discussion, but our conclusions will apply also to the chiral superfield $\mathcal{W}^\alpha \mathcal{W}_\alpha$, and hence to gaugino condensation.

It is important to note that a supersymmetric effective Lagrangian description of the form (1) will apply whether or not SUSY is spontaneously broken. As in the usual case of a spontaneously broken symmetry, the interactions themselves respect the symmetry, and it is only the ground state that breaks the symmetry. The relevant energy scale here is the scale at which SUSY is broken, so the superfield whose F-component acquires a vacuum expectation value will still appear in (1).

Now consider the vacuum expectation value of the F-component of one of the chiral superfields:

$$
\langle F_j^* \rangle \;=\; (g_{ij})^{-1} \, \frac{\partial W}{\partial \Phi_i} \;=\; (g_{ij})^{-1} W_{\Phi_i}.
\tag{3}
$$

(A similar equation holds for gauge-invariant chiral superfields formed by products of the Φ_i. It is perhaps more correct to use such fields as order parameters for SUSY breaking.) Supersymmetry will be broken if any of the $\langle F_i^* \rangle$ are non-zero. Suppose that this is established for some range of parameters (bare couping τ_0, masses m_i, etc.). This implies that the function W_{Φ_i} is non-zero in that range, and since the superpotential W depends holomorphically on these parameters, W_{Φ_i} cannot vanish outside the range except at some isolated points where it crosses through zero. However, modulo singularities in the Kähler metric, this implies that supersymmetry can only be restored at some isolated points in parameter space, and will generically remain broken as the parameters of the model are varied. (Here and below, by "point" we mean some subspace of lower dimensionality than the entire parameter space. Along the relevant direction orthogonal to this subspace the exceptional behavior will only occur at a point.)

We can also apply this line of reasoning to models which preserve supersymmtery in some patch of parameter space. In that case all $\langle F_i^* \rangle$ vanish in the patch, and hence everywhere by holomorphy. Exceptions again come from singularities in g_{ij}.

It is possible that the Kähler metric can become singular at certain points, for example when new exactly massless modes appear in the low energy theory. However, we expect that this again only occurs at an isolated point, and not in an entire patch of parameter space. Consider approaching the point g_* in parameter space where a new massless excitation (or multiplet of excitations) enters the low-energy theory. Near this special point, we should be able to write a new effective Lagrangian with the extra particle included. This new effective Lagrangian should have non-singular Kähler metric, and the mass of the new excitation should appear as a term in the superpotential. Now consider varying the relevant parameter past the point g_*. Since the particle mass is non-zero on one side of g_*, it cannot remain zero on the other side due to holomorphy. Hence, away from g_* there is no exactly massless excitation and therefore the original Kähler metric was only singular at g_* and not in an entire neighborhood. This is essentially what happens in $N = 2$ SQCD at the singularities in the complex u plane [9]. Of course, this discussion is in no way conclusive – we can not rule out the possibility of more pathological behavior of the Kähler metric. However, most analyses of SUSY dynamics concentrate on the superpotential, and assume that the Kähler metric is well-behaved. Under those assumptions one always finds the type of behavior that we emphasize in this note.

These conclusions are very similar to Seiberg's [10] observation that phase transitions are forbidden by holomorphy in SUSY models (again, modulo singularities in the Kähler

metric). Seiberg's argument focuses on the ground state energy

$$V \;=\; \sum_{ij} \, (g_{ji})^{-1} \, \frac{\partial W}{\partial \Phi_i} \frac{\partial W^*}{\partial \Phi_j^\dagger} \qquad\qquad (4)$$

which is also determined by a holomorphic function, up to singularities in the Kähler metric. Positivity of the vacuum energy and the smooth behavior of (4) are then enough to preclude phase transitions – although several phases can co-exist as degenerate vacua, the system will never make a discontinuous jump from one phase to another. We are merely making an analogous argument regarding F-component vacuum expectations which break SUSY dynamically. However, there is an important issue to discuss here, namely that models with softly-broken SUSY *can* exhibit phase transitions. As discussed in [11], softly-broken models (such as QCD) can be obtained from SUSY theories in the limit of large vacuum expectations for the F-components of spurion chiral superfields, with the superpotential remaining holomorphic in the spurion fields. We should try to understand what goes wrong with the usual arguments against phase transitions when the spurion F-components are turned on. In particular, what suddenly allows the existence of phase transitions in these models? Are there suddenly patches of singularities in the Kähler metric? The answer is actually more mundane. When spurion fields with non-zero F-components are present, terms in the Kähler potential can easily mimic terms in the superpotential. Equation (3) no longer applies, as there can be terms of the type

$$F_{\mathrm{spur}}^* F_i \;\; G(A_i^*, A_j) \qquad\qquad (5)$$

in the Lagrangian, where G is some undetetermined function and A_i are the lowest components of the superfields Φ_i. The effect of (5) on (4) is to shift the potential by some non-holomorphic quantity. At low energies ($E << F_{\mathrm{spur}}^*$), these terms can dominate the superpotential contributions, and can easily lead to phase transitions.

However, this loophole does not appear in our analysis of spontaneous SUSY breaking. All non-zero F-components in models with spontaneous SUSY breaking must be proportional to the same SUSY breaking scale – there is no frozen spurion field F_{spur}. In that case equation (3) always applies and a true singularity of the Kähler metric is required for non-analytic behavior. The type of behavior we predict for models with spontaneous SUSY breaking is therefore completely analogous to the absence of phase transitions in models with unbroken SUSY.

A possible application of these observations is to the construction of new models which break SUSY. Consider a model described by the bare Lagrangian $\mathcal{L}_0$ which breaks SUSY. Now consider adding some new interaction[1]: $\mathcal{L} = \mathcal{L}_0 + g\mathcal{O}$. In many cases it is easy to

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[1]This can be, for example, the gauging of some global symmetry already present in the model. Then we can explore SUSY breaking as a function of the new gauge coupling constant.

investigate the effect of adding the interaction $g\mathcal{O}$ when g is infinitesimal. For example, often the effect on the vacuum energy vanishes with g, and hence for sufficiently small g the vacuum energy will remain non-zero and SUSY will remain broken. But because SUSY is broken in the region g near zero, it will generically be broken for all values of g in the larger class of models. Interestingly, when g is large the mechanism for SUSY breaking may differ significantly from the original mechanism in $\mathcal{L}_0$. (See the discussion below of the $3-2$ model for an example of this.) Alternatively, in some cases adding the extra interaction $g\mathcal{O}$ will restore SUSY at non-zero g. In these cases the larger class of models is seen as generically SUSY preserving, with $g = 0$ an exceptional point of SUSY breaking.

Another possiblity is the addition of new, massive particles to the original model. In some cases it is possible to conclude that the addition of the new particles does not change the status of SUSY breaking if they are sufficiently heavy. In this case, one can conclude that the status of SUSY breaking remains the same as the mass of the heavy particles is reduced, and they become relevant to the low-energy dynamics.

In section 3 we will discuss some examples of the above behavior, including some new models of SUSY breaking which are constructed in this way.

2 Relation to the Witten index

It is worthwhile to compare our conclusions to those that can be obtained from the Witten index [8]. The Witten index is defined as

$$\mathrm{Tr}(-1)^{\mathrm{F}} \;=\; \mathrm{n}_0^{\mathrm{B}} \;-\; \mathrm{n}_0^{\mathrm{F}} \quad , \tag{6}$$

and counts the difference in the number of bosonic and fermionic zero modes. It is a consequence of the SUSY algebra that non-zero energy states come in degenerate fermion-boson pairs, and hence continuous deformations of a model cannot change $\mathrm{Tr}(-1)^{\mathrm{F}}$, except in the special case where a vacuum state comes in from or out to infinity along a pseudo-flat direction carrying a non-zero contribution to index.

When $\mathrm{Tr}(-1)^{\mathrm{F}} \neq 0$ SUSY is clearly preserved, and the implications are similar to what we concluded from holomorphy: generic variations of the parameters leave SUSY intact, except for some special values where a vacuum state escapes to infinity. However, the case of $\mathrm{Tr}(-1)^{\mathrm{F}} = 0$ is ambiguous, because there may either be no zero energy states (violating SUSY) or an even number of paired zero energy states (preserving SUSY). It is for vanishing Witten index that holomorphy provides some interesting new information. As a parameter in the model is changed, it is possible for a pair of eigenstates with non-zero energy to flow down

to zero energy, thereby restoring SUSY if it was initially broken. However, what we learn from the holomorphy argument is that this pair of eigenstates cannot "stick" at zero energy: the pair must instead merely touch zero at some special value of the parameter and then "bounce" back up to non-zero energy, or alternatively only reach zero energy asymptotically as the parameter approaches infinity. There is nothing wrong with non-analytic behavior of the eigenvalues, as the energy of a state (*e.g.* see equation (4)) is not itself holomorphic in the parameters[2]. On the other hand, if $\mathrm{Tr}(-1)^{\mathrm{F}} = 0$ but SUSY is preserved, we do not expect the zero energy pair of eigenvalues to leave $E = 0$ except at some special point in parameter space, where, *e.g.* , the SUSY vacuum escapes to infinity. In the next section we will see explicit examples of these types of behaviors.

3   Examples

Below we discuss some explicit examples of the general behavior described in the previous sections. Included are some new models of SUSY breaking constructed along previously described lines.

3.1   Quantum deformation of moduli space: $SU(2)$ + singlets

SUSY breaking can arise from the quantum deformation of moduli space [3, 4, 5]. A simple example of a moduli space with a quantum deformed constraint is $SU(2)$ with four doublet matter fields Q_i, $i = 1, \ldots, 4$. The classical moduli space is parameterized by the gauge invariants $M_{ij} = Q_i Q_j$ subject to the constraint Pf $M \equiv \epsilon^{ijkl} M_{ij} M_{kl} = 0$. Quantum mechanically, the constraint is modified to Pf $M = \Lambda_2^4$. While the point $M_{ij} = 0$ is part of the classical moduli space, it does not lie on the quantum moduli space.

Supersymmetry is broken if the quantum modification of the moduli space is incompatible with a stationary superpotential, $W_\phi \neq 0$. In the $SU(2)$ case, supersymmetry would be broken if there were F terms which only vanished for $M_{ij} = 0$. A simple realization of this [5] is to add to the $SU(2)$ model given above, six singlet fields $S^{ij} = -S^{ji}$, where

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[2]This is in contrast to quantum systems with a *finite* number of degrees of freedom (*e.g.* SUSY QM), where the groundstate energy is generally analytic in the parameters. Witten [8] pointed out that this implies that SUSY QM must behave in the manner advocated here for SUSY field theories. However, his argument does not obviously extend to field theory as in the limit of an infinite number of degrees of freedom analyticity can fail. (This is related to the well-known loophole that allows for phase transitions and symmetry breaking in the infinite volume limit.)

$i, j = 1, \ldots, 4$, with couplings

$$W_0 = \lambda \ S^{ij} Q_i Q_j = \lambda \ S^{ij} M_{ij}. \tag{7}$$

This superpotential leaves invariant an $SU(4)_F$ flavor symmetry under which Q_i transform as **4**, and S^{ij} transform as **6**. There is an anomaly free $U(1)_R$ symmetry under which $R(Q) = 0$ and $R(S) = 2$. Classically, there is a moduli space of supersymmetric vacua with $M_{ij} = 0$ and S^{ij} arbitrary. Quantum mechanically, the S^{ij} equations of motion, $\lambda M_{ij} = 0$, are incompatible with the quantum constraint Pf $M = \Lambda_2^4$. The classical moduli space of supersymmetric vacua is completely lifted for $\lambda \neq 0$ as a result of the quantum modification of the $SU(2)$ moduli space, and supersymmetry is broken.

Let us now consider some modifications of the theory. First, consider adding to the following interaction to the superpotential W_0:

$$W_1 \ = \ W_0 \ + \ g \epsilon_{ijkl} S^{ij} M^{kl} \ . \tag{8}$$

The new interaction, unlike W_0, is not invariant under $U(4)_F$ flavor rotations with determinant minus one. It thus violates a parity-like symmetry of the original model.

The condition that W_1 be extremal with respect to variations in S_{ij} now requires that

$$\lambda M_{ij} \ = \ - \ g \epsilon_{ijkl} M^{kl} \ . \tag{9}$$

Combining this with the quantum constraint and extremality of W_1 with respect to variations in M_{ij} then requires

$$\lambda^2 \ = \ 4g^2 \ . \tag{10}$$

We see that, for generic values of the new coupling g, SUSY remains broken, with the exception of the special points satisfying (10). From our previous discussion of the Witten index, we conclude that at these special points a pair of boson/fermion states drops to zero energy, maintaining $\mathrm{Tr}(-1)^{\mathrm{F}} = 0$ but restoring SUSY.

It is interesting to note[3] that one could rewrite the model in terms of shifted singlet fields

$$X_{ij} \ = \ \lambda S_{ij} \ + \ g \epsilon_{ijkl} S^{kl} \ . \tag{11}$$

Then the superpotential is simply $W_1 = X^{ij} M_{ij}$, and minimizing with respect to X_{ij} appears to break SUSY in the same manner as in the original model with W_0. This is correct for generic values of λ and g, but for those satisfying (10) the above conclusion fails as the transformation (11) becomes non–invertible. This leads to a singular Kähler metric in terms

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[3]We thank Y. Shirman for bringing this to our attention.

of X_{ij}, if one begins with a non-singular Kähler metric in S_{ij}. This example illustrates how SUSY can be restored at a special point due to a singularity in g_{ij}. It also shows the danger of assuming that the Kähler metric remains smooth while making arbitrary field redefinitions.

Alternatively, we can consider modifcations which explicitly violate $U(1)_R$ symmetry, enlarging the parameter space of the model. The low energy effective superpotential will then contain additional interactions, unconstrained by $U(1)_R$. For a generic superpotential of this type, analysis of the extremality conditions $W_\phi = 0$ shows that they can now be satisfied when any of the $U(1)_R$–violating couplings are non-zero. Thus in this larger space of models SUSY is generically preserved, and the $U(1)_R$–preserving point is exceptional. One can confirm that as the new couplings are taken to zero, the SUSY vacuum escapes to infinity.

Finally, we can consider the addition of additional massive quark flavors. For instance, add an additional pair of $SU(2)$ doublets, increasing the number of flavors to 3 (so $N_f = N_c + 1$). Then the M_{ij} satisfy the quantum constraint

$$(\text{Pf } M) \left(\frac{1}{M}\right)_{ij} = \Lambda_3^3 \, m_{ij}, \tag{12}$$

where Λ_3 is related to the strong scale of the 3-flavor model, and the indices now run from $1, ..., 6$. We will consider the case in which quarks $1 - 4$ remain massless and coupled to the singlet fields, while $5, 6$ are given the mass m. Then the constraint (12) requires $M_{i5} = M_{i6} = 0$, $i = 1, ..6$, while the light degrees of freedom satisfy a modified constraint $\text{Pf} M = \Lambda_3^3 m = \Lambda_2^4$. Thus, the low energy dynamics is essentially that of the $N_f = 2$ model, but with a modified strong coupling scale obtained by matching at the heavy quark mass. SUSY remains broken for all values of m, except at the special point $m = 0$, where again a pair of boson/fermion states must drop to zero energy.

3.2 $3 - 2$ model; gauging global U(1); massive matter

Another interesting example is provided by the 3-2 model [7, 5]. The matter content of the model is (here we indicate the $SU(3) \times SU(2)$ charges): $P\ (3,2)$, $L\ (1,2)$, $\overline{U}\ (\overline{3},1)$, $\overline{D}\ (\overline{3},1)$. This is just the one generation supersymmetric standard model without hypercharge, the positron, or Higgs bosons. Classically, this model has a moduli space parameterized by three invariants: $Z = P^2\overline{U}\overline{D}$, $X_1 = PL\overline{D}$, and $X_2 = PL\overline{U}$. There is another gauge invariant, $Y = P^3L$, which vanishes classically by Bose statistics of the underlying fields. The gauge group is completely broken for generic vacua on the classical moduli space; the above invariants are the fields which are left massless after the Higgs mechanism. At the bare level there is a single renormalizable coupling which can be added to the superpotential, $W_0 = \lambda X_1$. This

superpotential leaves invariant non-anomalous accidental $U(1)_R$ and $U(1)$ flavor symmetries, and completely lifts the classical moduli space. Classically, there is a supersymmetric ground state at the origin, with the gauge symmetries unbroken.

Non-perturbative gauge dynamics generate an additional term in the effective super-potential; the exact effective superpotential is fixed by holomorphy, symmetries, and an instanton calculation to be

$$W = \frac{\Lambda_3^7}{Z} + \mathcal{A}(Y - \Lambda_2^4) + \lambda X_1. \tag{13}$$

where $\mathcal{A}$ is a Lagrange multiplier field. The first term is generated by instantons in the broken $SU(3)$; it is the usual dynamical superpotential familiar from $SU(3)$ dynamics in the limit $\Lambda_3 >> \Lambda_2$. The second term enforces the quantum deformed constraint $Y = P^3 L = \Lambda_2^4$, which can be seen in the limit $\Lambda_2 >> \Lambda_3$. In this second limit we can neglect the $SU(3)$ dynamics and consider the $SU(2)$ theory which has two flavors and a quantum deformed constraint. For nonsingular Kähler potential, (13) lifts the classical ground state $Z = X_i = 0$. In the ground state of the quantum theory, both the $U(1)_R$ and supersymmetry are spontaneously broken.

There are several aspects of this model which illustrate our previous discussion:

• We can vary the bare coupling constants α_3, α_2. SUSY is broken generically for all values, but via different mechanisms in the two limits [5]. In the $\Lambda_2 >> \Lambda_3$ limit we have a quantum-deformed constraint, whereas in the opposite limit we have a dynamically generated superpotential.

• We can gauge the $U(1)$ flavor symmetry [2]. SUSY is clearly still broken for weak gauging, and must remain so even when the extra $U(1)$ is strongly coupled.

• Finally, we can add massive matter in vector like representations (section 6 of [5]). SUSY remains broken for generic values of the masses.

3.3   SU(5) with $\bar{5} + 10$

This model (along with $SO(10)$ with a single **16**) is one of the early candidate models for dynamical SUSY breaking [7], but is difficult to study because it is strongly coupled and has no flat directions.

Murayama [12] proposed a technique to study such models, by the introduction of extra vector like 5 and $\bar{5}$ fields with mass m, which produce pseudo-flat directions. He finds SUSY breaking for non-zero values of m, and suggests that this behavior must persist in the $m \to \infty$ limit, using the vanishing Witten index as justification. However, Murayama's analysis still

contains a potential loophole, as he originally noted. Zero Witten index does not preclude the energy of a pair of fermion/boson states from approaching zero at large m, restoring SUSY. (As happens in the models of section **3.1** at the special points $\lambda^2 = 4g^2$ or $m = 0$.) Unfortunately, even the holomorphy argument cannot quite close this loophole. It tells us that as $m \to \infty$, SUSY remains broken for generic m, but cannot exclude the possibility that SUSY breaking turns off asymptotically as $m \to \infty$.

Acknowledgments

References

[1] N. Seiberg, Phys. Lett. B 318 (1993) 469 ;
K. Intriligator, N. Seiberg, Nucl. Phys. Proc. Suppl. 45 BC (1996) 1.

[2] M. Dine, A.E. Nelson and Y. Shirman, Phys. Rev. D51 (1995) 1362.

[3] K. Intriligator, N. Seiberg and S.H. Shenker, Phys. Lett. B342 (1995) 152.

[4] K.-I. Izawa and T. Yanagida, Prog. Theor. Phys. 95 (1996) 829.

[5] K. Intriligator and S. Thomas, Nucl. Phys. B473 (1996) 121.

[6] C. Csaki, L. Randall and W. Skiba Nucl. Phys. B479 (1996) 65;
E. Poppitz and S. Trivedi, **hep-th/9609529**

[7] I. Affleck, M. Dine and N. Seiberg, Nucl. Phys. B241 (1984) 493;
I. Affleck, M. Dine and N. Seiberg, Nucl. Phys. B256 (1985) 557.

[8] E. Witten, Nucl. Phys. B202 (1982) 253.

[9] N. Seiberg and E. Witten, Nucl. Phys. B426 (1994) 19, erratum- Nucl. Phys. B430 (1994) 485.

[10] N. Seiberg and E. Witten, Nucl. Phys. B431 (1994) 484.

[11] N. Evans, S.D.H. Hsu, M. Schwetz, Phys. Lett. B355 (1995) 475;

[12] H. Murayama, Phys. Lett. B355 (1995) 187.